Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME: *LEERDERNAAM:*	Solno

GRADE 12 *GRAAD 12*	

NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

GRADE 12/*GRAAD 12*

JMATHE4

JUNE/*JUNIE* 2017

MATHEMATICS P2/*WISKUNDE V2*
SPECIAL ANSWER BOOK/
SPESIALE ANTWOORDEBOEK

QUESTION/*VRAAG*	MARK/*PUNT*	INITIAL/*PARAAF*	MOD.
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
TOTAL/*TOTAL*			

This SPECIAL ANSWER BOOK consists of 22 pages. /
Hierdie SPESIALE ANTWOORDEBOEK bestaan uit 22 bladsye.

QUESTION/*VRAAG* 1

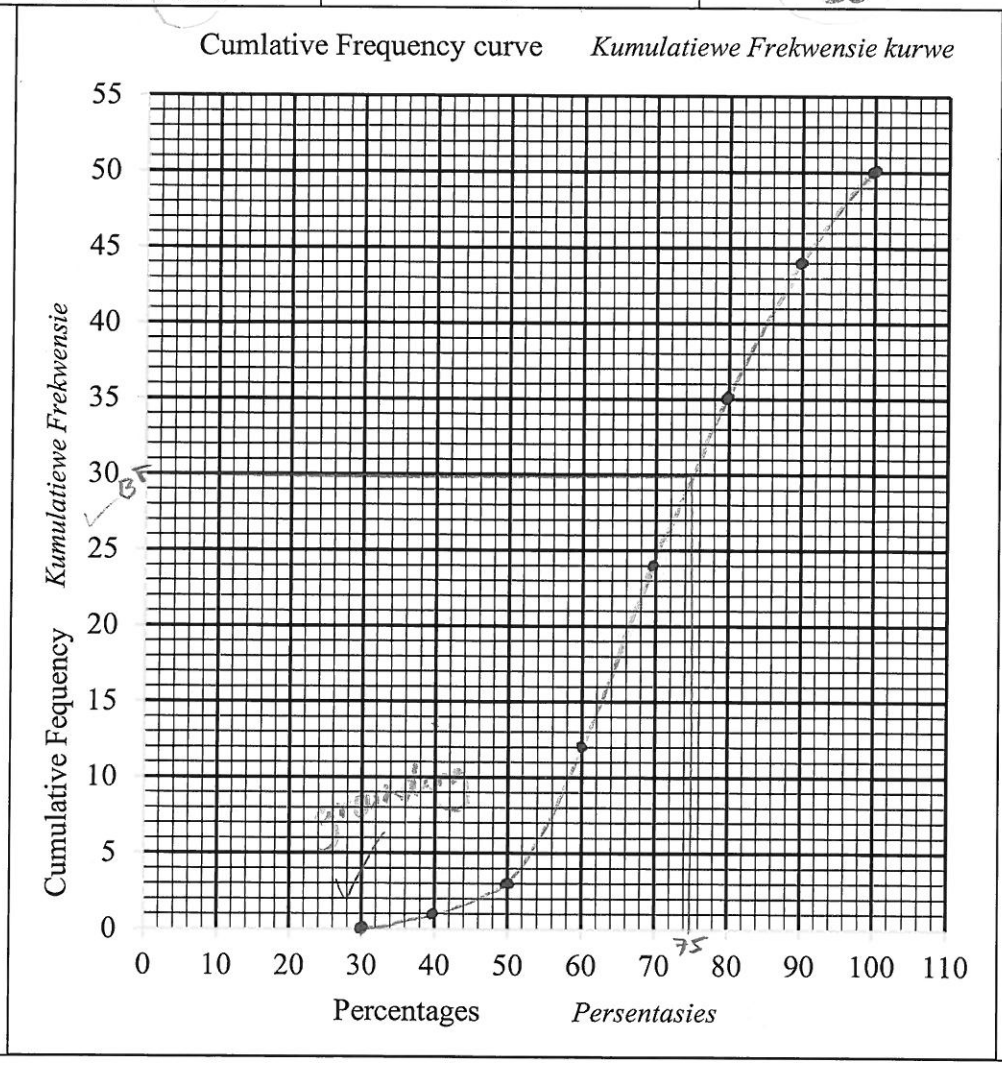
1.1

Percentages/ *Persentasies*	Frequency/ *Frekwensie*	Cumulative Frequency/ *Kumulatiewe Frekwensie*
$30 \leq x < 40$	1	1
$40 \leq x < 50$	2	3 ✓
$50 \leq x < 60$	9	12 ✓
$60 \leq x < 70$	12	24 ✓
$70 \leq x < 80$	11	35 ✓
$80 \leq x < 90$	9	44
$90 \leq x < 100$	6	50 ✓

3

(3)

1.2



✓ appro
✓ cum freq
✓ shape

4

(4)

1.3

$\leq$ 75 % = B

= 30 ✓ 28 – 32

2

(2)

[9]

1.	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12,4	15,1	18,9	19,7	19,7	20	20,9	23,7	23,8	31,1	33,6	34,5	34,9	36,5	40,1
			Q_1				M				Q_3			

QUESTION/*VRAAG* 2

2.1

min = 12,4

$Q_1 = 19,7$ ✓

$M = 23,7$ ✓ ✓

$Q_3 = 34,5$ ✓

max = 40,1

$T_1; \dots; T_{15}$

$M = T_{\frac{1}{2}(1+15)} = T_8$

$T_1; \dots; T_7$ $\quad T_9; \dots; T_{15}$

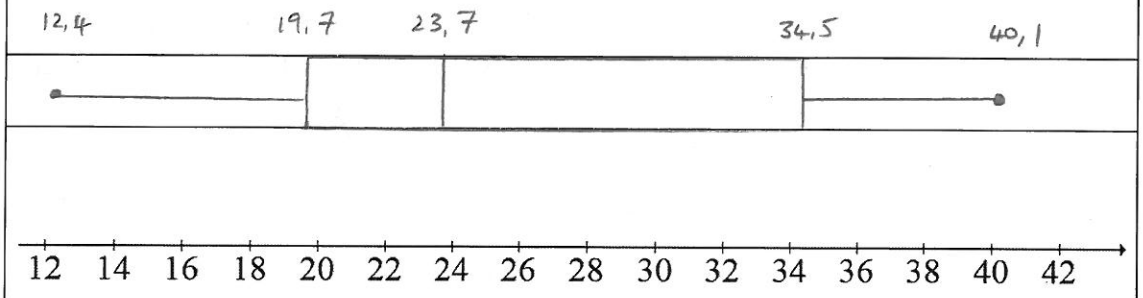
$Q_1 = T_{\frac{1}{2}(1+7)}$ $\quad Q_3 = T_{\frac{1}{2}(9+15)}$

$= T_4$ $\quad = T_{12}$

4

(4)

2.2



3

(3)

2.3

$M - Q_1 = 23,7 - 19,7$ $\quad Q_3 - M = 34,5 - 23,7$

$= 4$ $\quad = 10,8$

$\therefore Q_3 - M > M - Q_1$ ✓

$\therefore$ data skewed to the right (OR)

data is positively skewed

1

(1)

2.4

$\sigma = 8,36$ ✓✓

2

(2)

2.5

σ small - data not spread out much

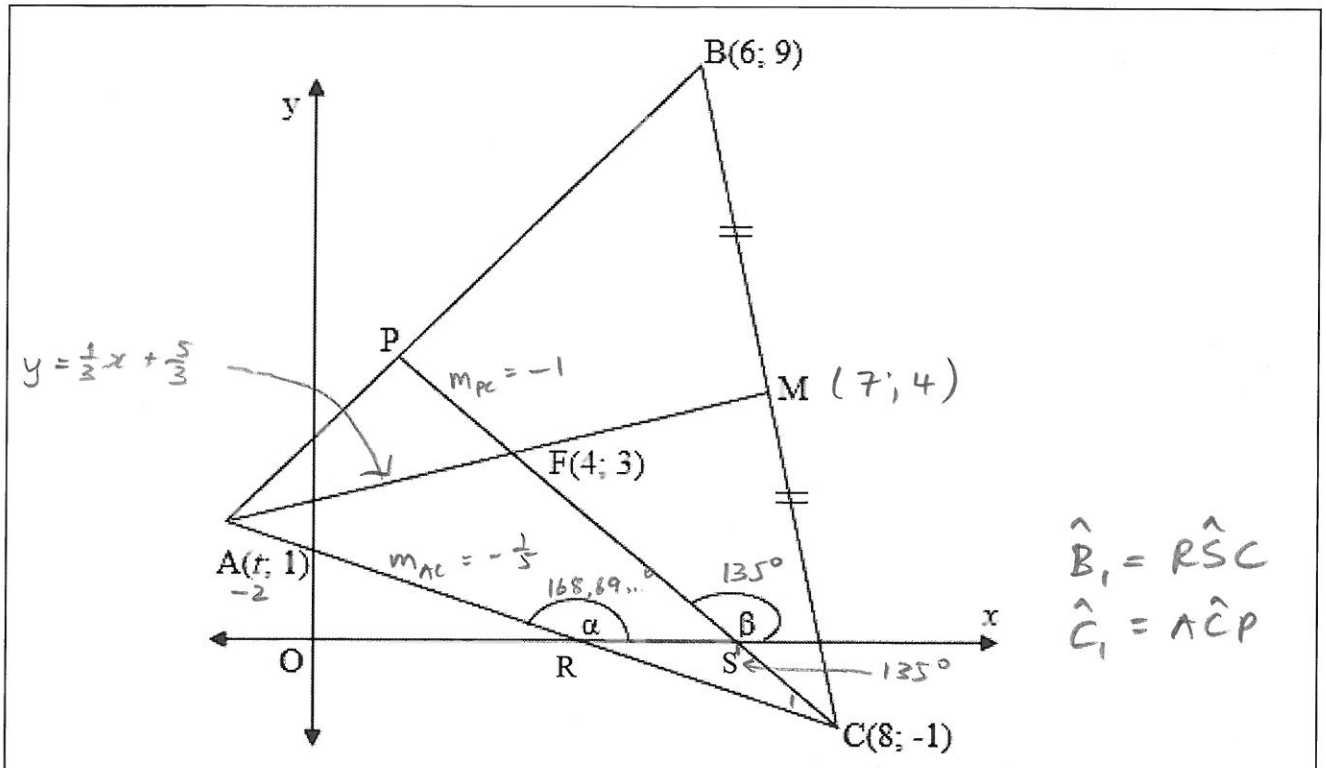
σ large - data very spread out ✓

1

(1)

[11]

QUESTION/*VRAAG* 3



3.1

$x_M = \frac{6+8}{2}$ $y_M = \frac{9+(-1)}{2}$ B (6; 9) C (8; -1)

$= 7$ $= 4$

$\therefore M(7; 4)$ ✓ ✓

2

(2)

3.2

$m_{AM} = \frac{4-3}{7-4} = \frac{1}{3}$ ✓ ✓ F (4; 3) M (7; 4)

$y = \frac{1}{3}x + c$

Sub F (4; 3)

$3 = \frac{1}{3}(4) + c$ ✓

$\frac{5}{3} = c$

$\therefore y = \frac{1}{3}x + \frac{5}{3}$ ✓

4

(4)

3.3

$y = \frac{1}{3}x + \frac{5}{3}$

sub $A(t;1)$

$1 = \frac{1}{3}(t) + \frac{5}{3}$ ✓

$-\frac{2}{3} = \frac{1}{3}t$

$-2 = t$ ✓ only (A) if −

(2) 2

3.4

$M_{PC} = \frac{-1-3}{8-4} = -1$ ✓ ✓ $F(4;3)$ $C(8;-1)$

(2) 2

3.5

$\tan\beta = m_{PC}$

$\tan\beta = -1$ ✓

ref $\angle = 45°$

tan − in

II: $\beta = 135°$ ✓

(2) 2

3.6

$m_{AC} = \frac{-1-1}{8-(-2)} = -\frac{1}{5}$ ✓ $A(-2;1)$ $C(8;-1)$

$\tan\alpha = -\frac{1}{5}$ ✓

ref $\angle = 11,30\ldots°$

tan − in

II: $\alpha = 168,69\ldots°$ ✓

$\hat{S}_1 = 135°$ vert opp $\angle$s =

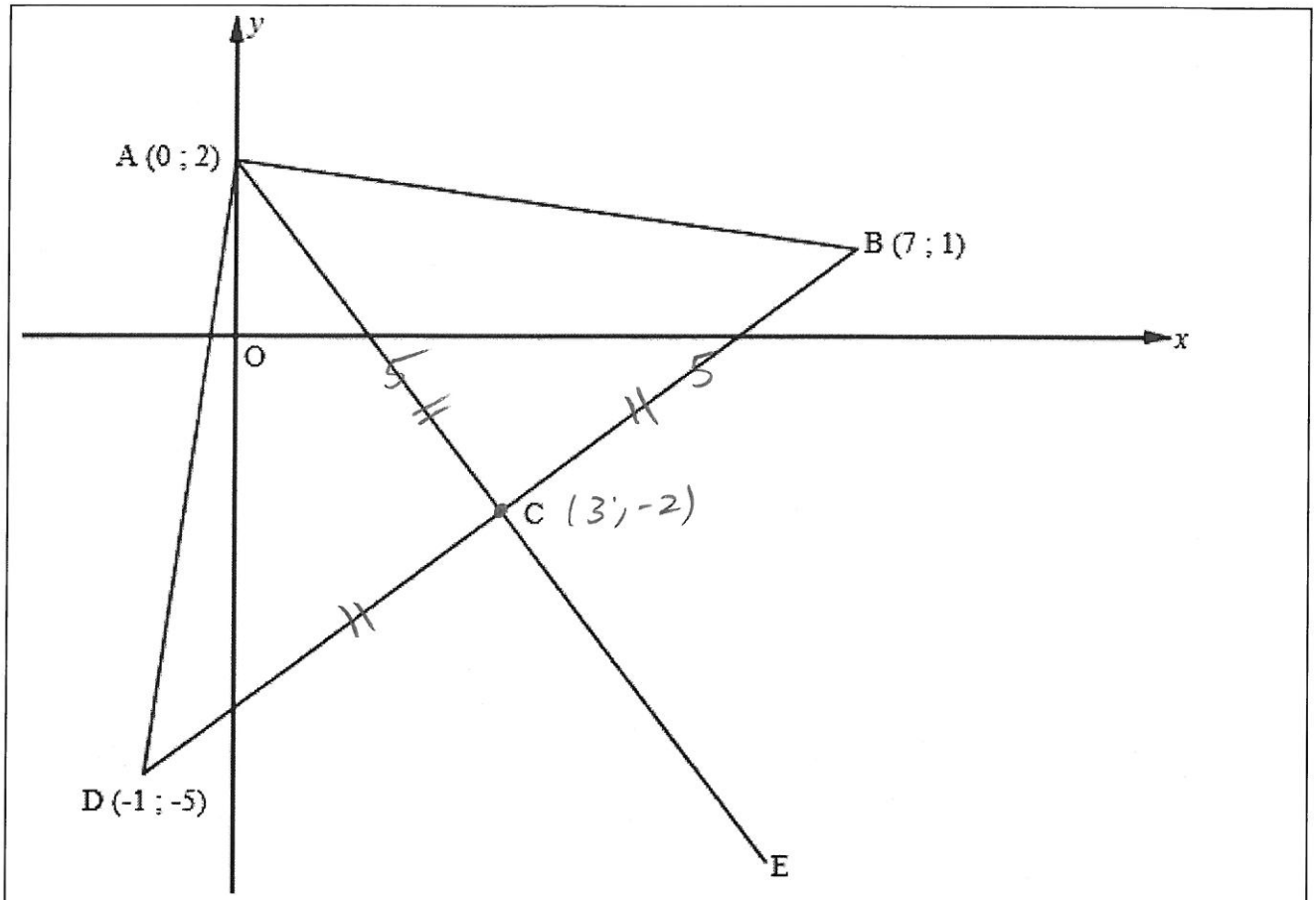
$135° + \hat{C}_1 = 168,69\ldots°$ ext $\angle\Delta$

$\hat{C}_1 = 33,69°$ ✓

(4) 4

[16]

QUESTION/*VRAAG* 4



4.1

$x_C = \frac{7+(-1)}{2}$ $y_C = \frac{1+(-5)}{2}$ B(7;1) D(-1;-5)

$= 3$ $= -2$

$\therefore$ C(3; -2) ✓ ✓

(2)

4.2

$CA = \sqrt{(2-(-2))^2 + (0-3)^2}$ ✓ C(3;-2) A(0;2)

$= \sqrt{25}$

$= 5$ ✓

$CB = \sqrt{(1-(-2))^2 + (7-3)^2}$ C(3;-2) B(7;1)

$= \sqrt{25}$

$= 5$ ✓

$\therefore CA = CB$ both = 5

(3)

4.3

$m_{AD} = \frac{-5-2}{-1-0} = 7$ ✓✓ A(0;2) D(-1;-5)

$m_{AB} = \frac{1-2}{7-0} = -\frac{1}{7}$ ✓✓ A(0;2) B(7;1)

$m_{AD} \times m_{AB} = 7 \times \left(-\frac{1}{7}\right)$ ✓

$= -1$

$\therefore AD \perp AB$

$\therefore D\hat{A}B = 90°$

(OR)

$AD = \sqrt{50}$ ✓S $AB = \sqrt{30}$ ✓S

$BD = 10$ ✓S

$\therefore DB^2 = AD^2 + AB^2$ ✓S

$\therefore D\hat{A}B = 90°$ converse Pyth ✓R

(5) 5

4.4

$(x-3)^2 + (y+2)^2 = 25$ ✓✓

$C(3;-2)$

$r = 5$

(2) 2

4.5

$m_{BC} = \frac{-2-1}{3-7} = \frac{3}{4}$ ✓✓

B(7;1) C(3;-2)

(2) 2

4.6

$y = -\frac{4}{3}x + c$ ✓ $m_{tan} = -\frac{4}{3}$ tan ⊥ rad

Sub B(7;1)

$1 = -\frac{4}{3}(7) + c$ ✓

$\frac{31}{3} = c$

$\therefore y = -\frac{4}{3}x + \frac{31}{3}$ ✓

(3) 3

4.7

$AC = CE$ and $DC = CB$ ⊙ ABED centre C (4;4)

$\therefore$ ABED is ||gm diags bisect

* but AE = DB diameters

(3) 3

$\therefore$ ABED is rectangle ||gm with diagonals = (4;3)

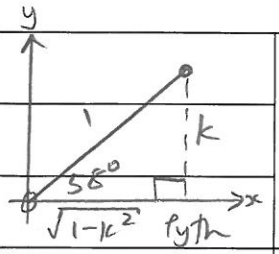
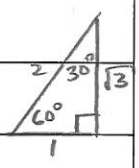
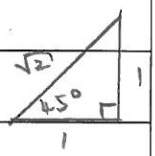
(OR) * but $D\hat{A}B = 90°$ ||gm with 1 int ∠ = 90°

$\therefore$ ABED is rectangle

[20]

QUESTION/*VRAAG* 5

$\sin 58^\circ = k = \frac{k}{1}$ $\frac{y}{r}$

5.1	5.1.1	$\sin 238^\circ = \sin(180^\circ + 58^\circ)$ $= -\sin 58^\circ$ ✓ $= -k$ ✓ 	(2) 2
	5.1.2	$\cos 58^\circ = \frac{\sqrt{1-k^2}}{1}$ $\frac{x}{r}$ ✓ diagram $= \sqrt{1-k^2}$ ✓ OR $\sin^2 58^\circ + \cos^2 58^\circ = 1$ $\therefore k^2 + \cos^2 58^\circ = 1 \quad \therefore \cos 58^\circ = \sqrt{1-k^2}$	(2) 2
5.2		• $\tan 150^\circ = \tan(180^\circ - 30^\circ) = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$ • $\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$  • $\sin 10^\circ$ • $\cos 225^\circ = \cos(180^\circ + 45^\circ) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$ • $\sin 135^\circ = \sin(180^\circ - 45^\circ) = \sin 45^\circ = \frac{1}{\sqrt{2}}$  • $\cos 80^\circ = \cos(90^\circ - 10^\circ) = \sin 10^\circ$ $\therefore \frac{(-\frac{1}{\sqrt{3}})(-\frac{\sqrt{3}}{2})\sin 10^\circ}{(-\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}})(\sin 10^\circ)} = \frac{\frac{1}{2}}{-\frac{1}{2}}$ ✓ values + triangles $= -1$ ✓	(7) 7
5.3		$\sin(\alpha+\beta) = \cos(90^\circ - (\alpha+\beta))$ ✓ $= \cos(90^\circ - \alpha - \beta)$ $= \cos((90^\circ - \alpha) - \beta)$ ✓ ✓ $= \cos(90^\circ-\alpha)\cos\beta + \sin(90^\circ-\alpha)\sin\beta$ ✓ $= \sin\alpha\cos\beta + \cos\alpha\sin\beta$	(4) 4

5.4

$LHS = \frac{\cos 2x + 1}{\sin 2x \tan x}$

$= \frac{2\cos^2 x - 1 + 1}{2\sin x \cos x \cdot \frac{\sin x}{\cos x}}$ ✓✓

$= \frac{2\cos^2 x}{2\sin^2 x}$

$= \frac{\cos^2 x}{\sin^2 x}$

$\therefore LHS = RHS$

$RHS = \frac{1}{\tan^2 x}$

$= \frac{1}{\frac{\sin^2 x}{\cos^2 x}}$

$= 1 \times \frac{\cos^2 x}{\sin^2 x}$

$= \frac{\cos^2 x}{\sin^2 x}$ ✓

4

(4)

5.5

5.5.1

$\tan x = 2\sin x$

$\frac{\sin x}{\cos x} - 2\sin x = 0$ ✓

$LCD = \cos x \ (\therefore \cos x \neq 0)$, x thru

$\sin x - 2\sin x \cos x = 0$ ✓

$\sin x (1 - 2\cos x) = 0$ ✓

$\therefore \sin x = 0$ or $\cos x = \frac{1}{2}$

3

(3)

5.5.2

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$x = k180°$ ✓

$Ref^{\angle} = 60°$

Cos + in

I: $x = 60° + k360°$ ✓

IV: $x = 300° + k360°$ ✓

$(k \in \mathbb{Z})$ ✓

4

(4)

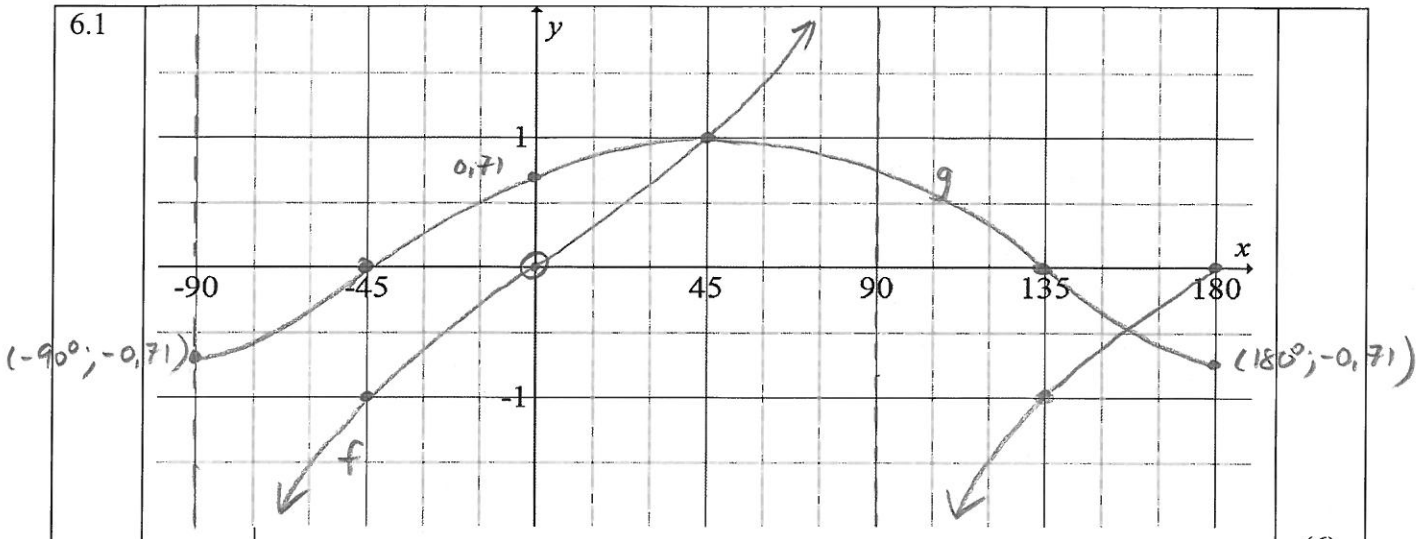
[26]

tan f g sin

asym ✓ ✓ shape and max (45°; 1)
x/y int ✓ ✓ x int
shape + arrowheads ✓ ✓ y int

QUESTION/*VRAAG* 6 6

6.1



(6)

6.2.1

$g(x) - f(x) = 1$

$y_g - y_f = 1$

* $x = -45°$ ✓✓

2

(2)

6.2.2

$g(x) \geq f(x)$

$y_g \geq y_f$

* $x \in (-90°; 45°]$

✓ values ✓ notn

2

(2)

6.3

$y = f(2x)$

$= \tan 2x$

New period $= \frac{180°}{2}$

$= 90°$ ✓

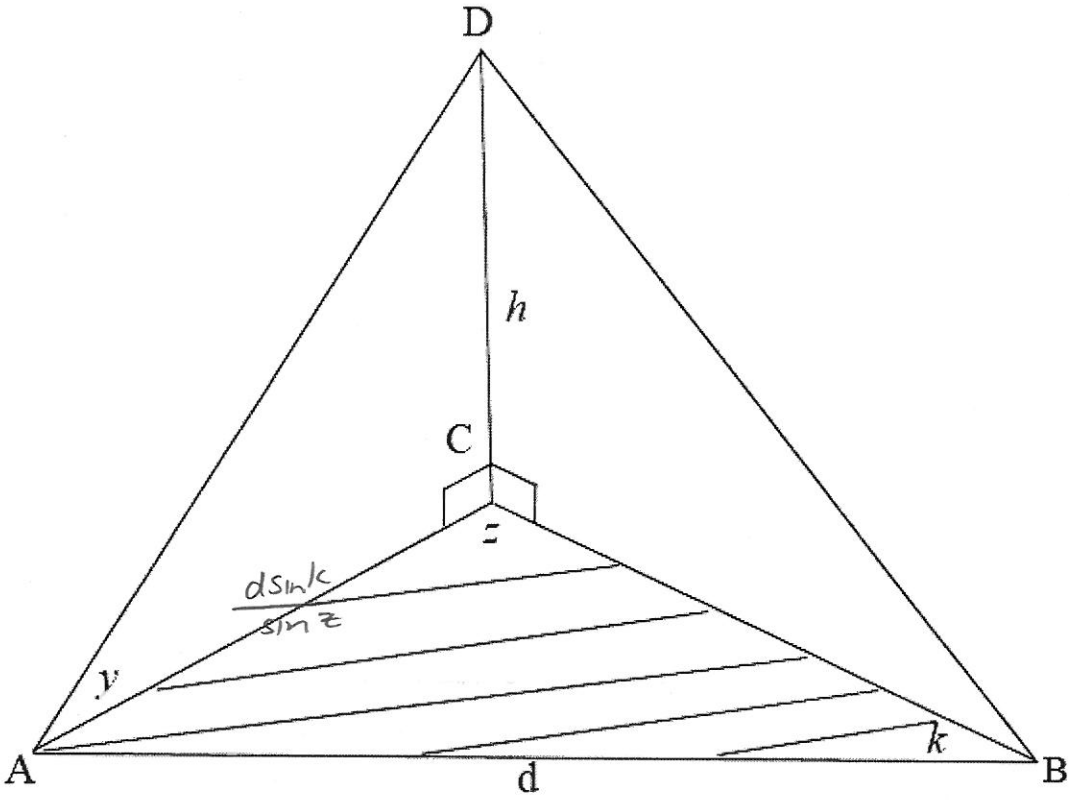
1

(1)

[11]

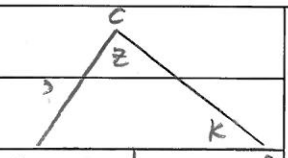
* penalise 1 mark each time if the restriction $x \in [-90°; 90°]$ is not adhered to.

QUESTION/*VRAAG* 7



7.1

$$\frac{AC}{\sin k} = \frac{d}{\sin z} \quad ✓$$



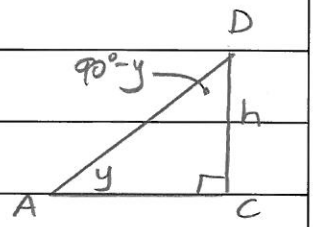
$$\therefore AC = \frac{d \sin k}{\sin z} \quad ✓$$

2

(2)

7.2

$$\tan y = \frac{h}{AC} \quad ✓$$



$$\therefore AC \cdot \tan y = h$$

$$AC = \frac{h}{\tan y} \quad ✓$$

2

OR

$$A\hat{D}C = 90° - y \qquad \text{^'s } \Delta = 180°$$

$$\frac{AC}{\sin(90° - y)} = \frac{h}{\sin y}$$

$$AC = \frac{h \cdot \cos y}{\sin y} \qquad * \text{ PTO}$$

(2)

7.3

$\frac{h}{\tan y} = \frac{d \sin k}{\sin z}$ ✓ both = AC

$\therefore h = \frac{d \sin k \tan y}{\sin z}$ ⟶

(1)

7.4

$h = \frac{d \sin k \tan y}{\sin z}$

$= \frac{80 \sin 38° \tan 40°}{\sin 125°}$ ✓

$= 50,45 \text{ m}$ ✓ ⟶

(2)

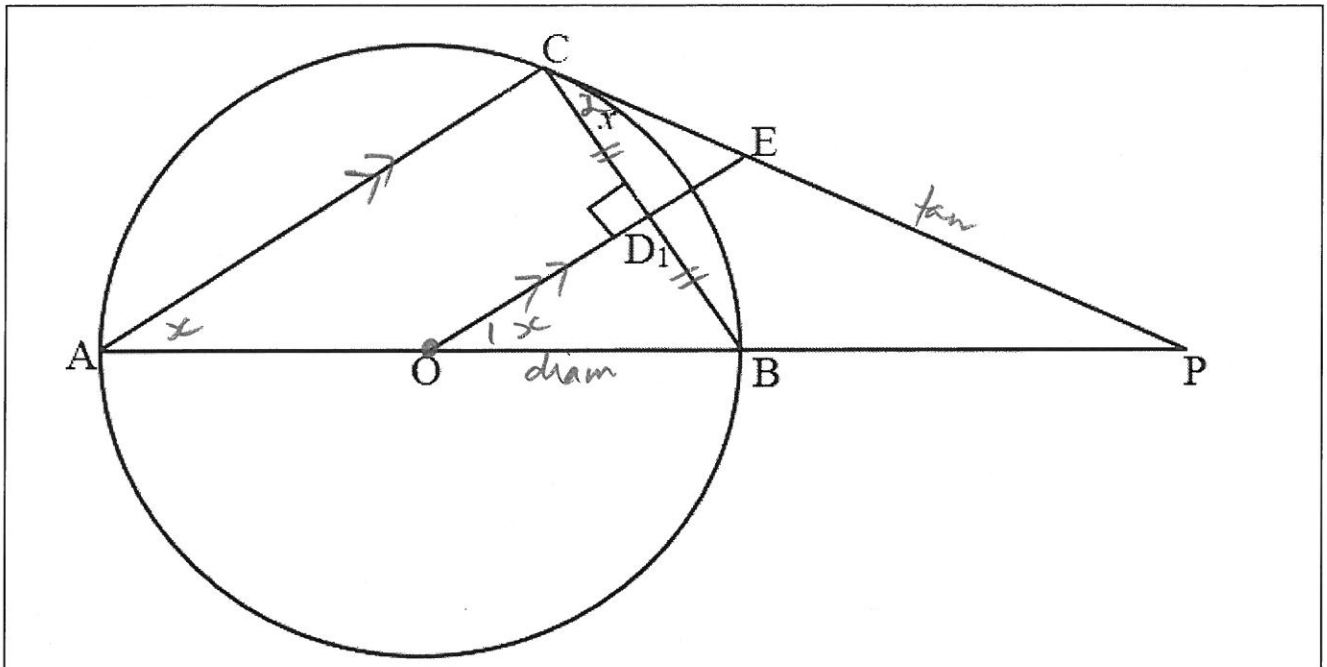
[7]

* $AC = h \cdot \frac{\cos y}{\sin y}$

$= h \div \frac{\sin y}{\cos y}$

$= \frac{h}{\tan y}$

QUESTION/*VRAAG* 8

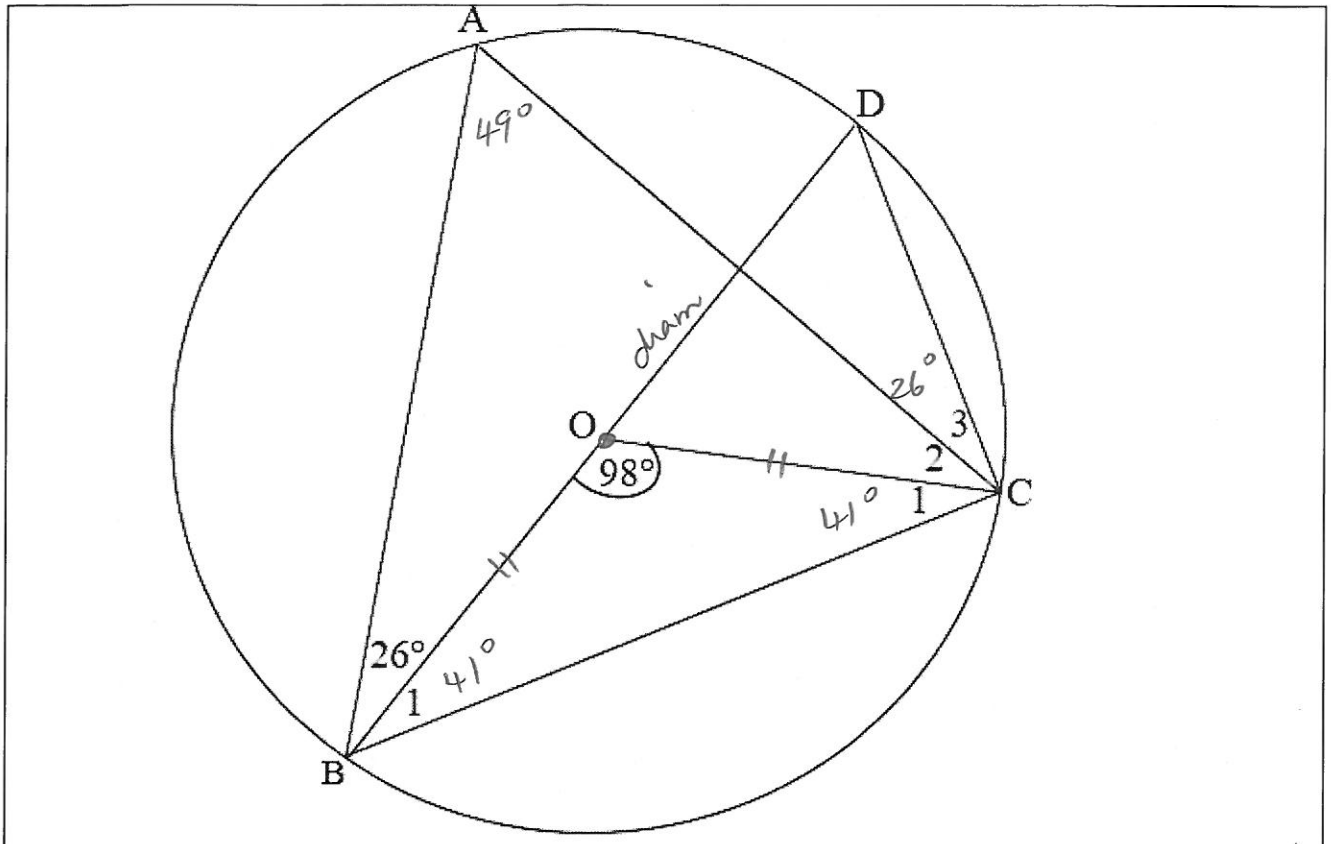


8.1	line from centre ⊙ ⊥ to chord ✓	(1)
8.2	$C\hat{D} = DB$ ✓ (8.1.) $AO = OB$ ✓ radii $\therefore OD \parallel AC$ ✓R midpt thm ⟹	(3)
8.3	$\hat{A} = x$ ✓S ✓R ∠ tan chord ⟹ $\hat{O}_1 = x$ ✓S ✓R Corr ∠'s =, $AC \parallel OE$ ⟹	(4)
8.4	$\hat{O}_1 = \hat{C}_2$ ✓S both $= x$ $\therefore OBEC$ is a ✓R Conv ∠'s in Same cyclic quad ⊙ segment = ⟹	(2)

[10]

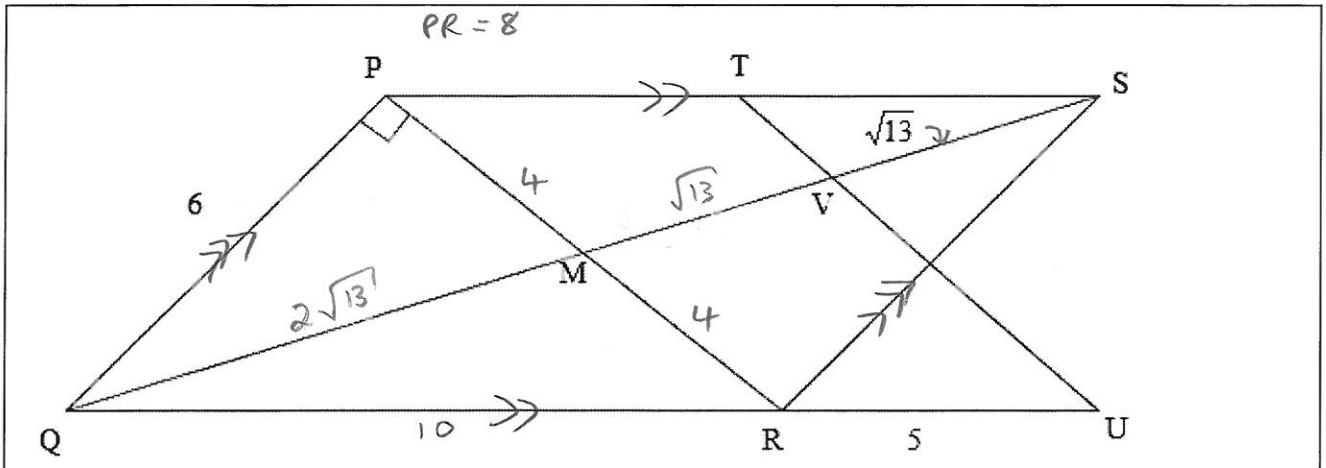
NB • PENALISE POORLY LABELLED SUBDIVIDED ANGLES
eg $A\hat{C}B$ vs $\hat{C}$ < $A\hat{C}B$? $A\hat{C}E$? $B\hat{C}E$?

QUESTION/*VRAAG* 9



9.1	$\hat{A} = 49°$ ✓S ✓R ^@ centre = 2 ^@ O'ce	(2)
9.2	$OB = OC$ radii $\hat{B}_1 = \hat{C}_1$ ✓R ^'s opp = sides $\therefore \hat{B}_1 = 41°$ ✓S ✓R ^'s $\Delta = 180°$	(3)
9.3	$\hat{C}_3 = 26°$ ✓SR ^'s in same ⊙ segment = $\hat{C}_2 = 23°$ ✓S ✓R ^ in semi ⊙ = 90°	(3)
		[8]

QUESTION/*VRAAG* 10



10.1 **10.1.1**

$RQ^2 = 6^2 + 8^2$ pythag ✓R

$RQ = 10$ ✓S

$\therefore \frac{UR}{RQ} = \frac{5}{10}$

$= \frac{1}{2}$ ✓ ⟹

(3)

10.1.2

$PM = 4$ ✓SR diags ||gm bisect

$\therefore MQ^2 = 6^2 + 4^2$ pythag

$\therefore MQ = 2\sqrt{13}$ $\sqrt{52}$

$\therefore MS = 2\sqrt{13}$ diags ||gm bisect

$\therefore MV = \sqrt{13}$ MS − VS

$\therefore \frac{VM}{MV} = \frac{\sqrt{13}}{2\sqrt{13}}$

$= \frac{1}{2}$ ⟹

(4)

10.2

$\frac{UR}{RQ} = \frac{5}{10} = \frac{1}{2}$ ✓S

$\therefore \frac{UR}{RQ} = \frac{VM}{MQ}$ both $= \frac{1}{2}$, (10.1.2)

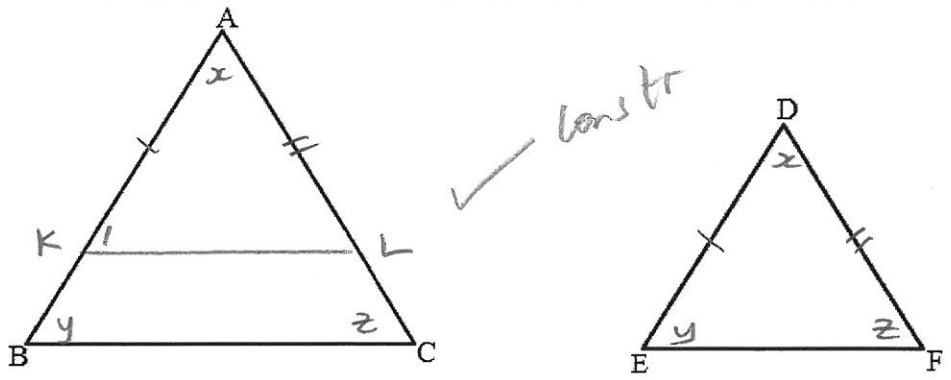
$\therefore MR \parallel VU$ ✓R line ÷ 2 sides of Δ in prop^n ⟹

(2)

[9]

QUESTION/*VRAAG* 11

11.1



Constr: as shown

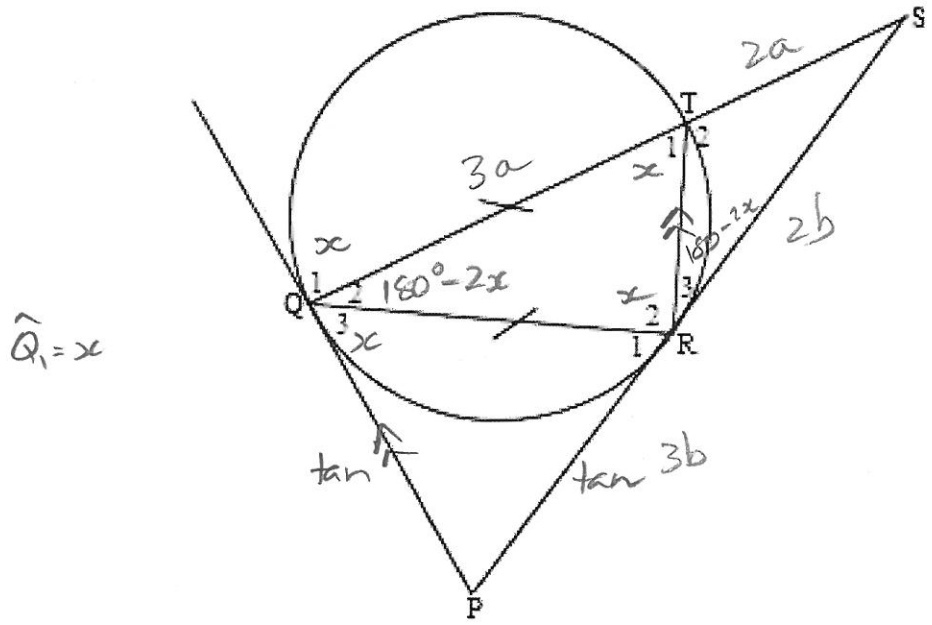
In Δ's AKL, DEF

1. AK = DE	constr	✓S
2. AL = DF	constr	
3. $\hat{A} = \hat{D}$	given	
$\therefore \Delta AKL \equiv \Delta DEF$	SAS	✓SR
$\therefore \hat{K}_1 = \hat{E}$	$\equiv \Delta$'s	
but $\hat{E} = \hat{B}$	given	
$\therefore \hat{K}_1 = \hat{B}$ ✓S	both $= \hat{E}$	
$\therefore$ KL ∥ BC ✓SR	corr $\angle$'s =	
$\therefore \frac{AB}{AK} = \frac{AC}{AL}$ ✓S ✓R	line ∥ side of Δ	
but AK = DE	constr	
and AL = DF	constr	
$\therefore \frac{AB}{DE} = \frac{AC}{DF}$		

7

(7)

11.2



11.2.1

$\hat{R}_2 = x$ ✓SR ∠ tan chord

$\hat{T}_1 = x$ ✓SR ∠'s opp = sides

$\hat{Q}_2 = x$ ✓SR ∠ tan chord

$(\hat{R}_1 = x)$

(3)

11.2.2

$\hat{Q}_2 = 180° - 2x$ ✓S ✓R ∠'s str line $= 180°$

(2)

11.2.3

$\hat{R}_2 = \hat{Q}_2$ ✓S ✓R both $= x$

$\therefore TR \parallel QP$ ✓R alt ∠'s =

(3)

11.2.4

In Δ's S T_2 R_3, S R_{2+3} Q_2

1. $\hat{S} = \hat{S}$ ✓SR common

2. $\hat{R}_3 = \hat{Q}_2$ ✓SR ∠ tan chord

$\therefore \Delta STR \,|||\, \Delta SRQ$ AAA ✓R

(3)

11.2.5

$\frac{SR}{SQ} = \frac{ST}{SR}$ ✓S ✓R $\Delta STR \;|||\; \Delta SRQ$

$\therefore SR^2 = ST \cdot SQ$ →

(2)

11.2.6

$\frac{QT}{TS} = \frac{3}{2}$

$\frac{SP}{PR} = \frac{5}{3}$ ✓SR line $\parallel$ 1 side of Δ

$PR = PQ$ ✓S ✓R tan's from ext common pt =

$\therefore \frac{SP}{PQ} = \frac{5}{3}$ →

(3)

[23]

TOTAL/*TOTAAL*: 150

Additional Space/*Addisionele Ruimte*